# GOD AND THE APPLICABILITY OF MATHEMATICS[1]

## *Introduction: Indispensability and Applicability of Mathematics*

One of the perennial and burning questions in the philosophy of mathematics concerns mathematics' indispensability and applicability. Though often conflated in the literature, these are actually distinct questions. Indispensability has to do with our inability to get along in science or in even ordinary life without quantifying over or using singular terms having as their referents mathematical objects. Applicability concerns mathematics' reliability or utility in helping us to navigate successfully the physical world.

Indispensability is thus first and foremost linguistic in character and only secondarily ontological. Most philosophers of mathematics acknowledge that our best scientific theories cannot be purged of quantification over, and singular terms for, mathematical objects; but many, if not most, will deny that we are thereby ontologically committed to the mind-independent reality of mathematical objects. Most of the philosophers who reject indispensability arguments for the reality of mathematical objects will be anti-realists about such objects, since these arguments are today the main motivation for the affirmation of mathematical realism, though even some realists may themselves eschew indispensability arguments in favor of other justifications of their realism. The indispensability of mathematics is, then, not widely contested, but the ontological implications of such indispensability is strongly contested across a wide variety of anti-realist perspectives.[2]

Applicability has to do with what mathematician and physicist Eugene Wigner famously called "the unreasonable effectiveness of mathematics in the natural sciences."[3] Mathematics is the language of nature. That is to say, the laws of nature may be expressed as mathematical equations which describe the phenomena to an astonishing degree of accuracy. In his important study *The Applicability of Mathematics as a Philosophical Problem* Mark Steiner has characterized the question raised by Wigner as a problem of *description*, to be differentiated from Steiner's own problem of *discovery*.[4] While the distinction is a helpful one, we must be careful not to attribute to Wigner a sort of literalism or naïve realism about the descriptive character of nature's laws. Wigner speaks rather of "an unexpectedly close and accurate description of the *phenomena*" and "the enormous *usefulness* of mathematics in the natural sciences."[5] As we shall see, he insists that nature's laws are useful and thus applicable even though they often cannot be taken as literal descriptions of the physical world. They accurately describe physical phenomena, and a multiplicity of such descriptions may be possible. Steiner, on the other hand, is concerned with the applicability of mathematics in the way in which mathematics serves as a means of scientific discovery. In this essay I shall be concerned only with Wigner's problem of the describability of the universe via mathematics rather than with its discoverability.

The applicability of mathematics is only indirectly related to the indispensability of mathematics. A Platonic realist like Peter van Inwagen does try to draw a connection between mathematics' applicability and indispensability by casting Platonic realism as at least a partial explanation of mathematics' applicability. He claims that, given the indispensability of mathematical terms and quantificational statements, arguments for even conclusions which are nominalistically acceptable, like "We shall need two gallons of paint for this room," are undermined by anti-realism, leaving us with no reason to think that truth should be ascribed to such conclusions. He writes,

> Anyone who denies the existence of numbers. . . must therefore regard the empirically verifiable fact that applying these principles to the physical world always yields the right result as a *mystery*. . . .
>
> Nominalism is therefore to be rejected because it renders the applicability of mathematics to the physical world a mystery.[6]

Realism presumably renders the applicability of mathematics non-mysterious.

In fact, however, van Inwagen's real target here is not anti-realism but fictionalism, the view that statements involving quantification over or reference to mathematical objects are, strictly speaking, false.[7] Fictionalism is but one of a cornucopia of anti-Platonic perspectives on mathematics, most of which affirm mathematical truth. Van Inwagen thus misleads in playing off Platonism solely against fictionalism, as though fictionalism were the only alternative to Platonic realism. The fundamental issue raised by fictionalism is mathematical truth, not the ontology of mathematical objects. The salient point made by van Inwagen is that we have good reason to think that many statements involving quantification over or reference to mathematical objects are true, in contradiction to fictionalism. It is only the conjunction of this conclusion with van Inwagen's neo-Quinean criterion of ontological commitment, which takes singular terms and the first-order existential quantifier to be devices of ontological commitment, that yields Platonism or, at least, realism.[8] Anti-realists who reject the neo-Quinean criterion may actually themselves press indispensability/applicability arguments for the truth of mathematical statements against their fictionalist colleagues without fear of ontological commitment to mathematical objects.

Our interest, then, is not in the indispensability of mathematics, nor in the ontological implications of such indispensability, but rather in the applicability of mathematics to the physical world, as evident from its utility in science.

## *The "Unreasonable Effectiveness of Mathematics"*

In his seminal paper Wigner makes two main points:

> (i) Mathematical concepts turn up in "entirely unexpected connections" in physics and often permit "an unexpectedly close and accurate description" of the phenomena in these connections.[9]

> (ii) Because we do not understand the reasons for the usefulness of mathematical concepts, we cannot know whether a scientific theory formulated in terms of such concepts is uniquely appropriate.

Wigner declines to develop (ii), which is not, in any case, germane to our interest.

## The Nature of Mathematical Inquiry

Turning to (i), Wigner first makes some preliminary remarks on the question, "What is mathematics?" which will prove to be relevant later. Here he stresses the *a priori* nature of mathematical inquiry, especially of the mathematics that is so valuable in physics:

> whereas it is unquestionably true that the concepts of elementary mathematics and particularly elementary geometry were formulated to describe entities which are directly suggested by the actual world, the same does not seem to be true of the more advanced concepts, in particular the concepts which play such an important role in physics. . . . Most more advanced mathematical concepts, such as complex numbers, algebras, linear operators, Borel sets - and this list could be continued almost indefinitely - were so devised that they are apt subjects on which the mathematician can demonstrate his ingenuity and sense of formal beauty.[10]

Wigner's point is well-taken. As philosopher of mathematics Penelope Maddy emphasizes, what justifies the use of various set-theoretical axioms by the mathematician is fruitfulness: axioms are properly adopted which are rich in mathematical consequences, or what Maddy calls "mathematical depth."[11] This fact is important because set theory is typically regarded as foundational for the rest of mathematics, since the whole of mathematics can be reductively analyzed in terms of pure sets.[12] Whereas Wigner represents the mathematician as inventing new concepts outside his axioms, Maddy represents him as inventing new axioms. Mathematicians are at liberty to craft and explore different axiomatic systems at will.

So set theorists have felt free to formulate quite a variety of set theories, some featuring even different mathematical objects. Maddy observes that mathematicians employ "maximizing principles of a sort quite unlike anything that turns up in the practice of natural science: crudely, the scientist posits only those entities without which she cannot account for our observations, while the set theorist posits as many entities as she can, short of inconsistency."[13] Maddy identifies quite a few of these "rules of thumb" followed by set theorists in choosing their axioms and constructing their theories, such as *maximize, richness, diversity, one step back from disaster*, *etc*.[14] Similarly Wigner observes, "The great mathematician fully, almost ruthlessly, exploits the domain of permissible reasoning and skirts the impermissible."[15]

The "principal point" which will be relevant to the uncanny effectiveness of mathematics is that mathematicians are not bound by customary axiomatic concepts but are free to define new concepts with a view, not of applicability or scientific utility, but of "permitting ingenious logical operations which appeal to our aesthetic sense both as operations and also in their results of great generality and simplicity."[16] That historically there has been a cross-pollination between physics and mathematics, physics sometimes spurring developments in mathematics,[17] does not nullify Wigner's point. Wigner finds a particularly striking example in complex numbers.

> Certainly, nothing in our experience suggests the introduction of these quantities. Indeed, if a mathematician is asked to justify his interest in complex numbers, he will point, with some indignation, to the many beautiful theorems in the theory of equations, of power series, and of analytic functions in general, which owe their origin to the introduction of complex numbers. The mathematician is not willing to give up his interest in these most beautiful accomplishments of his genius.[18]

Who, then, would have anticipated the centrality and utility of complex numbers in physical theory?[19]

## Mathematics' Role in Physics

Wigner now inquires as to the role of mathematics in physics and why mathematics' success in that role appears "so baffling." With respect to mathematics' role in physics, Wigner notes that while mathematics is useful in physics for evaluating the consequences of the laws of nature, a role which he associates with applied mathematics, it also plays a more "important" and "sovereign" role in physics, namely, to enable the formulation of the laws of nature themselves in the language of mathematics in order to be an apt object for the use of applied mathematics. By laws of nature Wigner understands statements of various regularities of the inanimate world which are conditional and limited in scope.[20]

To illustrate the importance of mathematical concepts in the formulation of the laws of physics, Wigner turns to Paul Dirac's formulation of the axioms of quantum mechanics. Wigner identifies two basic concepts in Dirac's formulation: vectors in Hilbert space, a peculiar, infinite-dimensional, mathematical space, and mathematical operators which act on these vectors to give the real (that is, neither complex nor imaginary) value of various observables or measurable quantities like position, momentum, spin, and so forth.[21] So we have two foundational mathematical concepts at play in Dirac's formulation of the laws of quantum mechanics: vectors in Hilbert space and special operators on those vectors. Wigner pulls up at this point "lest we engage in a listing of the mathematical concepts developed in the theory of linear operators."[22]

Wigner rejects the suggestion that the physicist had to choose this particular formulation of quantum mechanics because of its simplicity. He reminds us that the Hilbert space of quantum mechanics is a complex space and that complex numbers are far from simple and cannot be suggested by physical observations. Moreover, the use of complex numbers in this case is not a calculational trick of applied mathematics but comes close to being a necessity in the formulation of the laws of quantum mechanics. It now appears, Wigner comments, that not only complex numbers but so-called analytic functions are destined to play a decisive role in the formulation of quantum theory.[23] Thus, mathematical concepts are of decisive importance in the formulation of the laws of quantum theory and cannot be regarded as forced upon us by simplicity.

At this point Wigner muses, “It is difficult to avoid the impression that a miracle confronts us here.”[24] The closest thing to an explanation of mathematical concepts’ cropping up in physics, Wigner reflects, is Einstein’s assertion that the only physical theories which we are willing to accept are the beautiful ones and, hence, the mathematically formulated ones, since mathematical concepts have the quality of beauty. Wigner rightly rejects this suggestion, since it explains at best why the theories we are willing to believe are mathematically formulated, not why the accurate, that is, empirically applicable, theories are mathematically formulated.

Thus far, Wigner has argued merely that mathematics plays a central role in the formulation of successful laws of physics, a conclusion which no one, I think, would contest. This conclusion is reinforced by the indispensability of mathematics for physical theories. The key question comes in the next section of Wigner’s paper, “Is the Success of Physical Theories Truly Surprising?”

## Is the Success of Physical Theories Truly Surprising?

In this section Wigner argues that “the mathematical formulation of the physicist’s often crude experience leads in an uncanny number of cases to an amazingly accurate description of a large class of phenomena.”[25] He provides three examples in support.

The first example is Newton’s second law of motion, $F = ma$ (where $F$ is the force on an object, $m$ is the mass of that object, and $a$ is the acceleration of the object). Newton’s law proved applicable, not merely to mundane objects, but to planetary motion. According to Wigner, the law though simple to the mathematician, is not simple for the untrained because a second derivative, namely, acceleration (change in change-in-displacement over time) appears in it. “Newton’s law, quoted over and over again, must be mentioned first as a monumental example of a law, formulated in terms which appear simple to the mathematician, which has proved accurate beyond all reasonable expectations.”[26] Wigner alludes as well to Newton’s gravitational law $F = G(m_1m_2/r^2)$,[27] which is algebraically related to the second law and has also been confirmed to a fantastic degree of accuracy. Newton’s gravitational equation was eventually superseded by Einstein’s ten equations for the gravitational field, which may be expressed as $R_{\mu\nu} - ½Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G/c^4 T_{\mu\nu}$.[28] The General Theory of Relativity based on the Einstein equations affords even more accurate predictions than Newton’s theory, solving, for example, the mystery of Mercury’s orbit, which Newton’s theory had failed precisely to predict.

Wigner’s second example is ordinary, elementary quantum mechanics. At the instigation of Max Born, who had noted specific similarities between the behavior of matrices and some of Werner Heisenberg’s earlier quantum work,[29] Heisenberg replaced the position and momentum variables in his equations with matrices and applied the rules of matrix mechanics to a few highly idealized problems, with satisfactory results. "The miracle occurred only when matrix mechanics, or a mathematically equivalent theory, was applied to problems for which Heisenberg's calculating rules were meaningless."[30] Heisenberg's original rules presupposed that the equations of classical mechanics had solutions with certain periodicity properties, that is to say, solutions which exhibit a regular, repetitive, wavelike pattern. But the equations of motion for the electrons of the helium atom, not to mention the electrons of heavier atoms, did not have these same properties. Therefore, Heisenberg's rules could not be applied to them. Nevertheless, alternative theories that utilized the ideas of matrix mechanics yielded calculations for the helium atom which Wigner says had an accuracy of "one part in ten million." A disagreement would have provoked a crisis in atomic physics. Wigner reflects, "physics as we know it today would not be possible without a constant recurrence of miracles similar to the one of the helium atom, which is perhaps the most striking miracle that has occurred in the course of the development of elementary quantum mechanics, but by far not the only one."[31]

Wigner's third example is quantum electrodynamics, a theory which unites quantum mechanics and relativity to describe electromagnetism,, and in particular the Lamb Shift, which described a small difference of about 0.00003% in the energy levels of the hydrogen atom not predicted by Dirac's equations. The theory is one of the most accurate ever devised, predicting the value of the fine structure constant $\alpha$ to an accuracy of $10^{-8}$.

Wigner notes that his three examples represent an increasing independence of empirical experience in favor of reliance on mathematics: "Whereas Newton's theory of gravitation still had obvious connections with experience, experience entered the formulation of matrix mechanics only in the refined or sublimated form of Heisenberg's prescriptions. The quantum theory of the Lamb shift. . . is a purely mathematical theory and the only direct contribution of experiment was to show the existence of a measurable effect."[32]

Wigner takes his examples—"which could be multiplied almost indefinitely"—to illustrate the "appropriateness" and "almost fantastic accuracy" of the mathematical formulation of the laws of nature in terms of mathematical concepts chosen for their "manipulability," that is to say, their "amenability to clever manipulations and to striking, brilliant arguments."[33]

## Reconstruction of Wigner's Argument

Steiner, without disputing the legitimacy of Wigner's mystery of nature's mathematical describability, does fault him for his "flawed presentation," which has hindered philosophers from giving him his due. Steiner reconstructs Wigner's argument as follows:[34]

1. Mathematical concepts arise from the aesthetic impulse in humans.

> 2. It is unreasonable to expect that what arises from the aesthetic impulse in humans should be significantly effective in physics.
>
> 3. Nevertheless, a significant number of these concepts are significantly effective in physics.
>
> 4. Hence, mathematical concepts are unreasonably effective in physics.

Steiner thinks that this formulation is problematic for two reasons: (i) It ignores the failures, that is, the instances in which scientists fail to find appropriate mathematical descriptions of natural phenomena, which outnumber the successes by far, as well as the mathematical concepts that have never found an application. (ii) Each individual success of applying a mathematical statement might have nothing to do with its being a *mathematical* concept.

I must confess that I fail, despite my best effort, to see the force of these alleged shortcomings. With respect to (i), the fact that scientists often fail to find mathematical laws to describe physical phenomena (in biology, for example) does nothing to nullify the fact that many such phenomena, especially in physics, are so describable, as Wigner illustrates, and so cry out for explanation. Moreover, it is to be expected that many of the infinitude of mathematical concepts will not be physically applicable, nor does Wigner's argument suggest otherwise.

As for (ii), Steiner later expands on the perceived flaw. Wigner, he says,

> gives persuasive examples of successes that cry out for explanation--but he doesn't prove that they add up to one phenomenon that cries out for explanation. Each success is a story in itself, which may or may not have an explanation. Wigner does not make a case that what is unreasonably effective is *mathematics*, even though the individual examples he gives are of concepts that happen to be mathematical. In other words Wigner may give examples of a number of applications which are 'unreasonably effective' –applications of concepts which happen to be mathematical. But he doesn't show that these successes have anything to do with the fact that the concepts are mathematical.[35]

This is an odd complaint. Steiner grants that each of the examples of applicability that Wigner gives is mathematical and that each "is so extraordinary that it requires explanation."[36] To complain that Wigner's argument focuses on isolated examples is to suggest that his examples may not be representative. But of what? Of physics? Wigner justifiably maintains that examples such as he gives pervade physics. Of mathematics? Wigner has no interest in showing that physical applicability pervades mathematics, nor should he. *Pace* Steiner, Wigner does *not* want to show that "these successes have [some]thing to do with the fact that the concepts are mathematical." The examples of successful application are not to be explained by the concepts' being mathematical; indeed, in Wigner's view mathematical concepts successfully apply *despite* their being mathematical. The concepts' being mathematical contributes, not to their successful application, but to the inexplicability of their successful application. For Wigner there is no explanation for their successful application, and so we are left with mystery. To refute Wigner one would need to show either that the relevant concepts, despite their mathematical nature, do have some explanation of their applicability, or else that the examples of inexplicable applicability are truly exceptions and that most of the successful applications of mathematics in physics are explicable.[37]

In any case Steiner seems to regard his alleged shortcomings of Wigner's argument as merely a flaw of presentation. Appealing to physicists who maintain that mathematical concepts as a whole require explanation for their applicability, Steiner says that this is "a separate question" and "I believe the most profound. It concerns the applicability of mathematics as such, not as this or that concept. . . . It is the question raised by Eugene Wigner."[38] Since the focus on isolated examples is merely a flaw of presentation, "Wigner's thesis is not in fact vulnerable to the objection that one is ignoring the evidence of failure."[39]

Furthermore, Steiner's reconstruction of Wigner's argument, which is nowhere explicitly formulated by Wigner, seems somewhat maladroit. For it follows from (1) and (2) that

> 4*. It is unreasonable to expect that mathematical concepts should be significantly effective in physics.

which is practically synonymous with (4). Yet this makes (3) superfluous to Wigner's argument, even though it represents the heart of his paper! Worse, (3) gives one the premiss for a *modus tolens* argument from (2) against (1).

We can avoid denying (1) if we take (2) to be an implicit conditional. (2) is plausibly true only on the assumption of metaphysical naturalism, the view that the physical world is all there is.[40] Accordingly, we could revise (2) to

> 2*. If naturalism is true, then it is unreasonable to expect that what arises from the aesthetic impulse in humans should be significantly effective in physics.

Then (3) would lead to a rejection of naturalism. Wigner, however, does not argue for theism but rests with mystery. Accordingly, we should take (2) to mean something like

> 2**. It would be surprising to find that what arises from the aesthetic impulse in humans should be significantly effective in physics.

Recall that this section of the paper asks "Is the Success of Physical Theories Truly Surprising?", not "Is the Success of Physical Theories Truly Unreasonable?" When, therefore, we learn that mathematical concepts are significantly effective, it occasions a mystery, something meriting explanation.[41]

Steiner's reconstruction also misses out completely on Wigner's strong emphasis on the laws of nature as mathematical descriptions of the phenomena.[42] Wigner is not concerned with how many mathematical concepts, for example, arithmetic concepts, are significantly effective in physics. He is concerned with nature's laws. Accordingly, (3) ought to be re-formulated as something along the lines of

> 3*. The laws of nature can be formulated as mathematical descriptions (concepts) which are often significantly effective in physics.

Wigner rightly emphasizes that this is a pervasive feature of the laws of physics. That scientists often fail in their fumbling attempts to discover or formulate nature's laws does nothing to undercut (3*). (Such botched efforts would again seem more relevant to Steiner's discoverability argument.) That an indefinite number of mathematical concepts fail to find application in the universe is to be expected, given the infinitude of the one and the finitude of the other, and does nothing to undercut the truth of (3*). To refute Wigner one would need to show that scarcely any physical phenomena are covered by laws of nature or that the relevant natural law is not mathematically formulable. While that may be the case in some fields of science, it does not seem to be the case in physics.

Steiner's formulation of (1) is unobjectionable, so long as we keep in mind that Wigner is not talking about aesthetics in the artistic sense, but in the sense of mathematical beauty, what Maddy calls mathematical depth. Mathematics is an a priori discipline which is independent of the physical world. Moreover, when we reflect that mathematical objects, even if they exist, are causally effete, it is surprising that such objects should be significantly effective in physics.[43] Indeed, the abstractness of mathematical objects would serve to explain in general terms, not just in isolated examples, why their applicability is so surprising.

Accordingly, the following seems to be a more suitable formulation of Wigner's argument:

> 1*. Mathematical concepts arise from the aesthetic impulse in humans and have no causal connection to the physical world.
>
> 2**. It would be surprising to find that what arises from the aesthetic impulse in humans and has no causal connection to the physical world should be significantly effective in physics.
>
> [Therefore, it would be surprising to find that mathematical concepts should be significantly effective in physics.]
>
> 3*. The laws of nature can be formulated as mathematical descriptions (concepts) which are often significantly effective in physics.

> 4**. Therefore, it is surprising that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics.

Given that something surprising merits *prima facie* an explanation, we wonder as to the explanation of the fact that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics.

### *Accounting for Mathematics' Applicability*

Wigner, despite his characterization of the applicability of mathematics to the physical world as a miracle, in the end regarded it as a mystery. He concluded, "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve."[44] Wigner, however, never actually considered in his essay whether theism might not furnish a good explanation of mathematics' applicability. He considered at most naturalistic explanations of it and, finding none to be satisfactory, therefore concluded "that the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it."[45] But suppose we take the theistic hypothesis seriously. Since the question of mathematics' applicability to the physical world is already a metaphysical, not a physical, question, there can be no objection stemming from the corner of methodological naturalism to considering a metaphysical answer to a metaphysical question.

Theists will have a considerably easier time, I think, explaining the applicability of mathematics than will naturalists. Theists hold that there is a personal, transcendent being (a.k.a. God) who is the Creator and Designer of the universe. Naturalists hold that all that exists concretely is space-time and its physical contents. Now whether one is a realist or an anti-realist about mathematical objects, it appears that the theist enjoys a considerable advantage over the naturalist in explaining the uncanny success of mathematics.

### Realism: Non-theistic and Theistic

Consider first realism's take on the applicability of mathematics to the world. For the *non-theistic* realist, the fact that physical reality behaves in accord with the dictates of acausal mathematical entities existing beyond space and time is, in the words of philosopher of mathematics Mary Leng, "a happy coincidence."[46] For consider: If, *per impossibile*, all the abstract objects in the mathematical realm were to disappear overnight, there would be no effect on the physical world. This is simply to underscore the fact that abstract objects are causally inert. The idea that realism somehow accounts for the applicability of mathematics "is actually very counterintuitive," muses Mark Balaguer. "The idea here is that in order to believe that the physical world has the nature that empirical science assigns to it, I have to believe that there are causally inert mathematical objects, existing outside of spacetime," an idea which is inherently implausible.[47]

It might be said that the applicability of mathematics is not puzzling because there are unlimited realms of mathematics which have no applicability whatsoever to the physical world. So it is hardly surprising that out of this infinite range of options, the physicist will find some mathematics to describe the laws of nature. Wigner seemed to anticipate this sort of response when he acknowledged,

> It is true, of course, that physics chooses certain mathematical concepts for the formulation of the laws of nature, and surely only a fraction of all mathematical concepts is used in physics. It is true also that the concepts which were chosen were not selected arbitrarily from a listing of mathematical terms but were developed, in many if not most cases, independently by the physicist and recognized then as having been conceived before by the mathematician. It is not true, however, as is so often stated, that this had to happen because mathematics uses the simplest possible concepts and these were bound to occur in any formalism.[48]

As Wigner sees, the question is not, why are there mathematical concepts and structures which are applicable to physical reality, for the question is not about the fecundity of the mathematical realm. Quite the reverse, the question is why the physical world exhibits a structure that is so amenable to mathematical description of its natural laws. As Wigner states, there is nothing about the mathematical formalism discerned in the laws of nature that renders its instantiation inevitable.[49]

By contrast, the *theistic* realist can argue that God has fashioned the world on the structure of the mathematical objects He has chosen. This is essentially the view that Plato defended in his dialogue *Timaeus*. Plato draws a fundamental distinction between the realm of static being (that which ever is) and the realm of temporal becoming (that which is ever becoming). The former realm is to be grasped by the intellect, whereas the latter is perceived by the senses. The realm of becoming is comprised primarily of physical objects, while the static realm of being is comprised of logical and mathematical objects. God looks to the realm of mathematical objects and models the world on it. The world has its mathematical structure as a result. Plato writes,

> We must in my opinion begin by distinguishing between that which always is and never becomes from that which is always becoming but never is. The one is apprehensible by intelligence with the aid of reasoning, being eternally the same, the other is the object of opinion and irrational sensation, coming to be and ceasing to be, but never fully real. . . . Whenever, therefore, the maker of anything keeps his eye on the eternally unchanging and uses it as his pattern for the form and function of his product the result must be good; whenever he looks to something that has come to be and uses a model that has come to be, the result is not good.
>
> . . . If the world is beautiful and its maker good, clearly he had his eye on the eternal; if the alternative (which it is blasphemy even to mention) is true, on

> that which is subject to change. Clearly, of course, he had his eye on the eternal; for the world is the fairest of all things that have come into being and he is the best of causes. That being so, it must have been constructed on the pattern of what is apprehensible by reason and understanding and eternally unchanging; from which again it follows that the world is a likeness of something else. . . .
>
> . . . For god's purpose was to use as his model the highest and most completely perfect of intelligible things, and so he created a single visible living being, containing within itself all living beings of the same natural order.[50]

Thus, the realist who is a theist has a considerable advantage over the naturalistic realist in explaining why mathematics is so effective is describing the physical world.

The main objection confronting this view is theological: the realm of mathematical objects is thought to exist independently of God, so that God is not the sole ultimate reality. There are on the contemporary scene Christian realists who limit God's creation to Plato's realm of temporal becoming and exempt the intelligible realm from creation.[51] But other Christian realists construe mathematical and other putative abstract objects to be in fact concrete objects, namely, thoughts of various sorts in the mind of God and so dependent upon God for their being.[52] Still others advocate absolute creation, the view that the realm of abstract objects, including mathematical objects, though necessary in its existence, is nonetheless causally dependent upon God.[53]

## Anti-realism: Non-theistic and Theistic

Now consider anti-realism of a *non-theistic* sort. It might be said by certain thinkers of post-modernist bent that mathematics is merely a projection of the human mind, and so it is hardly surprising that physicists should concoct the mathematical concepts and structures they need for their theories. Again Wigner seemed to anticipate such a response:

> A possible explanation of the physicist's use of mathematics to formulate his laws of nature is that he is a somewhat irresponsible person. As a result, when he finds a connection between two quantities which resembles a connection well-known from mathematics, he will jump at the conclusion that the connection is that discussed in mathematics simply because he does not know of any other similar connection. It is not the intention of the present discussion to refute the charge that the physicist is a somewhat irresponsible person. Perhaps he is. However, it is important to point out that the mathematical formulation of the physicist's often crude experience leads in an uncanny number of cases to an amazingly accurate description of a large class of phenomena.[54]

Whatever other failings a post-modern view of mathematical truth might have, the salient point here is that the amazing accuracy of physics in a vast range of cases is most plausibly explained by there being an objectively existing physical world which one discovers to be amenable to mathematical description. It is irrelevant whether the mathematical realm is a mere pretence. The world really operates in accord with mathematically formulated laws; this is not plausibly an illusion of human consciousness.

Leng, on the other hand, says that on anti-realism relations which are said to obtain among pretended mathematical objects just mirror the relations obtaining among things in the world, so that there is no happy coincidence. Philosopher of physics Tim Maudlin muses, "The deep question of why a given mathematical object should be an effective tool for representing physical structure admits of at least one clear answer: because the physical world literally has the mathematical structure; the physical world is, in a certain sense, a mathematical object."[55]

Well and good, but what remains wanting on naturalistic anti-realism is an explanation *why* the physical world should exhibit so elegant and stunning a mathematical structure in the first place. After all, there is no necessity that a physical world exist at all, in which case mathematical truths would not have been descriptive of the physical world. Perhaps the universe, in order to exist, had to have *some* mathematical structure--though couldn't the world have been a structureless chaos?[56]--but that structure might have been describable by elementary arithmetic. For example, one thing and another thing make two things. But, as Wigner is at pains to emphasize, modern physics shows the physical world to be breathtakingly mathematically elegant. When Einstein was struggling to craft his General Theory of Relativity, for example, he had first to go to a mathematician to be tutored in tensor calculus before he could advance further to formulate his equations of the gravitational field. The laws of nature are contingent, at least in the sense that the states of affairs described by them did not have to obtain. By using as his examples laws of nature which are fearsomely complicated mathematically, Wigner already forced the question to a higher plane.

Not only so, but by choosing examples like the infinite-dimensional Hilbert space and complex numbers, Wigner implicitly precluded the explanation that physical reality is isomorphous to such mathematical structures, since these cannot be physically realized in the universe. According to Steiner, physicists see no difficulty in the applicability of arithmetic to the world, since this is just a matter of logic, not physics; rather they concentrate upon the seemingly miraculous appropriateness of physically meaningless concepts like matrix algebra or Hilbert spaces for quantum mechanics.[57] It is the burden of Steiner's book to provide numerous examples of the applicability of mathematical concepts that cannot be physically instantiated.[58] Some of his examples are the same ones to which Wigner already appealed, such as the descriptive applicability of analytic functions of complex variables, Heisenberg's utilization of matrix mechanics in his classical equations, a procedure for which, Steiner says, "there is no physical rationale" and which replaces all the variables by matrices "which have no physical meaning," and the descriptive applicability of the Hilbert space formalism to quantum mechanics, which Steiner calls "physically unintelligible." So even if the physical universe had to have some mathematical structure, that fails to address the question raised by Wigner. In the end Balaguer admits that he has no explanation why, on anti-realism, mathematics is applicable to the physical world or why it is indispensable in empirical science. He just observes that neither can the realist answer such "why" questions.

By contrast, the *theistic* anti-realist has a ready explanation of the applicability of mathematics to the physical world: God has created it according to a certain blueprint which He had in mind. There are any number of blueprints He might have chosen. Maddy observes,

> contemporary pure mathematics works in application by providing the empirical scientist with a wide range of abstract tools; the scientist uses these as models—of a cannon ball's path or the electromagnetic field or curved spacetime—which he takes to resemble the physical phenomena in some rough ways, to depart from it in others. . . . The applied mathematician labors to understand the idealizations, simplifications and approximations involved in these deployments of his abstract structures; he strives as best he can to show how and why a given model resembles the world closely enough for the particular purposes at hand. In all this, the scientist never asserts the existence of the abstract model; he simply holds that the world is like the model is some respects, not in others. For this, the model need only be well-described, just as one might illuminate a given social situation by comparing it to an imaginary or mythological one, marking the similarities and dissimilarities.[59]

On theistic anti-realism the laws of nature have the mathematical form they do because God has chosen to create the world according to the abstract model He had in mind. This was the view of the first century Jewish philosopher Philo of Alexandria, who maintained in his treatise *On the Creation of the World* that God created the physical world on the mental model in His mind. For a Jewish monotheist like Philo, the realm of Ideas does not exist, as Plato thought, independently of God but as the contents of His mind. Philo referred to the mind of God as God's Logos (Word). The sensible world (*kosmos oratos*) is made on the model of the conceptual or intelligible world (*kosmos noētos*) that pre-exists in the Logos. Philo explains:

> God, because He is God, understood in advance that a fair copy would not come into existence apart from a fair model, and that none of the objects of sense-perception would be without fault, unless it was modeled on the archetypal and intelligible idea. When he had decided to construct this visible cosmos, he first marked out the intelligible cosmos, so that he could use it as a incorporeal and most god-like paradigm and so produce the corporeal cosmos, a younger likeness of an older model, which would contain as many sense-perceptible kinds as there were intelligible kinds in that other one.
>
> To declare or suppose that the cosmos composed of the ideas exists in some place is not permissible. How it has been constituted we will understand if we pay careful attention to an image drawn from our own world. When a city is founded, in accordance with the high ambition of a king or a ruler who has laid claim to supreme power and, because he is at the same time magnificent in his

> conception, adds further adornment to his good fortune, it can happen that a trained architect comes forward. Having observed both the favourable climate and location of the site, he first designs in his mind a plan of virtually all the parts of the city that is to be completed—temples, gymnasia, public offices, market-places, harbours, shipyards, streets, construction of walls, the establishment of other buildings both private and public. Then, taking up the imprints of each object in his own soul like in wax, he carries around the intelligible city as an image in his head. Summoning up the images by means of his innate power of memory and engraving their features even more distinctly in his mind, he begins, like a good builder, to construct the city out of stones and timber, looking at the model and ensuring that the corporeal objects correspond to each of the incorporeal ideas.
>
> The conception we have concerning God must be similar to this, namely that when he had decided to found the great cosmic city, he first conceived its outlines. Out of these he composed the intelligible cosmos, which served him as a model when he also completed the sense-perceptible cosmos. Just as the city that was marked out beforehand in the architect had no location outside, but had been engraved in the soul of the craftsman, in the same way the cosmos composed of the ideas would have no other place than the divine Logos who gives these (ideas) their ordered disposition. After all, what other place would there be for his powers sufficient to receive and contain, I do not speak about all of them, but just a single one of them in its unmixed state? If you would wish to use a formulation that has been stripped down to essentials, you might say that the intelligible cosmos is nothing else than the Logos of God as He is actually engaged in making the cosmos. For the intelligible city too is nothing else than the reasoning of the architect has he is actually engaged in the planning the foundation of the city.[60]

Especially noteworthy is Philo's insistence that the world of ideas cannot exist anywhere but in the divine Logos. Just as the ideal architectural plan of a city exists only in the mind of the architect, so the world of ideas exists solely in the mind of God. Philo's view can be interpreted as either a conceptualist realism or as an anti-realism, for he says that the intelligible world may be thought of as either formed by the divine Logos or, more reductively, as the Logos itself as God is engaged in creating. Since Philo believed that time had a beginning at creation, the formation of the intelligible realm in the divine mind should probably be thought of as timeless and as explanatorily prior to God's creation of the sensible realm.

## *Conclusion*

Thus, the theist—whether he be a realist or an anti-realist about mathematical objects—has the explanatory resources to account for the otherwise unreasonable effectiveness of mathematics in physical science—resources which the naturalist lacks.

We may thus extend Wigner's argument:

1*. Mathematical concepts arise from the aesthetic impulse in humans and have no causal connection to the physical world.

2**. It would be surprising to find that what arises from the aesthetic impulse in humans and has no causal connection to the physical world should be significantly effective in physics.

[Therefore, it would be surprising to find that mathematical concepts should be significantly effective in physics.]

3*. The laws of nature can be formulated as mathematical descriptions (concepts) which are often significantly effective in physics.

4**. Therefore, it is surprising that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics.

5. Therefore, the fact that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics merits explanation.

6. Theism provides a better explanation of the fact that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics than does atheism.

7. Therefore, the fact that the laws of nature can be formulated as mathematical descriptions that are often significantly effective in physics provides evidence for theism.

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[1] I am grateful to David Hutchings, David Sherrill, and Matthew Probert for their valuable input to this essay.

[2] For a survey of anti-realist and arealist perspectives as well as non-Platonic realisms, see my *God and Abstract Objects: The Coherence of Theism III: Aseity* (Berlin: Springer Verlag, 2017).

[3] Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," in *Communications in Pure and Applied Mathematics* 13/1 (New York: John Wiley & Sons, 1960).

[4] Mark Steiner, *The Applicability of Mathematics as a Philosophical Problem* (Cambridge, Mass.: Harvard University Press, 1998), p. 9. Steiner's project is to show that physicists' use of mathematical analogies to guess at the laws of nature is anthropocentric, that is, it presupposes that human beings have a special importance in

the scheme of things. The stunning success of such a procedure, attested by numerous examples, calls into question naturalism, which holds that the universe is indifferent to the goals and values of humanity (Ibid., pp. 3-8; cf. pp. 55, 176). Steiner contends that "If we examine the analogies actually used to discover the major laws of physics in our century, we find that the analogies used are anthropocentric. On naturalist grounds, then, they should have failed, just as a dowser should fail to find oil. And this is a difficulty for naturalism, because what the evidence suggests is, on the contrary, that nature looks 'user-friendly' to human inquiry" (Ibid., p. 72). Not only is Steiner's project compatible with Wigner's, but Steiner recognizes that his conclusion is consistent with natural theology as well (Ibid., p. 10).

[5] Wigner, "Unreasonable Effectiveness of Mathematics" [my emphasis], p. 2.

[6] Peter van Inwagen, "Fictionalist Nominalism and Applied Mathematics," *The Monist* 97/4 (2014): 486; *cf*. pp. 495-6. The "principles" to which he here alludes are correspondence principles correlating adjectival use of numerals with nominal use of numerals, which are employed as premises to obtain a true, nominalistically acceptable conclusion. Since the truth of the conclusion is not guaranteed by the truth of the premises, he wants to know, why does mathematics work?

[7] For an account of fictionalism, see Mark Balaguer, "Fictionalism in the Philosophy of Mathematics," in *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta (Stanford University, 1997–). Article published September 21, 2013. http://plato.stanford.edu/archives/fall2013/entries/fictionalism-mathematics/; Mark Balaguer, *Platonism and Anti-Platonism in Mathematics* (New York: Oxford University Press, 1998).

[8] On the neo-Quinean criterion of ontological commitment, see Mark Balaguer, "Platonism in Metaphysics," in *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta (Stanford University, 1997–). Article published April 7, 2009. http://plato.stanford.edu/archives/sum2009/entries/platonism/.

[9] Wigner, "Unreasonable Effectiveness of Mathematics," p. 2. Wigner re-phrases this first point as "mathematics plays an unreasonably important role in physics."

[10] Ibid., pp. 2-3.

[11] Penelope Maddy, *Defending the Axioms: On the Philosophical Foundations of Set Theory* (Oxford: Oxford University Press, 2011), p. 82.

[12] See Penelope Maddy, *Naturalism in Mathematics* (Oxford: Clarendon Press, 1997), chapter 2, "Set Theory as a Foundation." Maddy comments,

"The astounding achievement of the foundational studies of the late nineteenth and early twentieth centuries was the discovery that these fundamental assumptions could themselves be proved from a standpoint more fundamental still, that of the theory of sets. The idea is simple: the objects of any branch of classical mathematics—numbers, functions, spaces, algebraic structures—can be modeled as sets, and resulting versions of the standard theorems can be proved in set theory. So the most fundamental of the fundamental assumptions of mathematics, the only such assumptions that truly cannot be proved, are the axioms of the theory of sets itself.

In this sense, then, our much-valued mathematical knowledge rests on two supports: inexorable deductive logic, the stuff of proof, and the set theoretic axioms" (Ibid., p. 1).

[13] Ibid., p. 131.

[14] Penelope Maddy, "Believing the Axioms I," *Journal of Symbolic Logic* 53/2 (1988): 481-511.

[15] Wigner, "Unreasonable Effectiveness of Mathematics," p. 3.

[16] Ibid.

[17] Ivor Grattan-Guiness, "Solving Wigner's Mystery: The Reasonable (Though Perhaps Limited) Effectiveness of Mathematics in the Natural Sciences," *Mathematical Intelligencer* 30/3 (2008): 7-17.

[18] Ibid.

[19] See now the possible relevance of so-called octonions, two steps beyond complex numbers (Natalie Wolchovern, "The Peculiar Math that Could Underlie the Laws of Nature," *Quanta Magazine* (July 28, 2018) https://www.wired.com/story/the-peculiar-math-that-could-underlie-the-laws-of-nature/.

[20] He summarizes, "All the laws of nature are conditional statements which permit a prediction of some future events on the basis of the knowledge of the present, except that some aspects of the present state of the world, in practice the overwhelming majority of the determinants of the present state of the world, are irrelevant from the point of view of the prediction" (Wigner, "Unreasonable Effectiveness of Mathematics," p. 5). For a good discussion of the nature of nature's laws see Jeffrey Koperski, *Divine Action, Determinism, and the Laws of Nature* (London: Routledge, 2020), chap. 5.

[21] Wigner refers to these as self-adjoint operators. They are also known as Hermitian operators. Self-adjoint operators are those which guarantee real values as an outcome. The presence of such an operator tells us that its corresponding quantity can

indeed be measured—hence, Wigner's calling the operators themselves "observables" in his paper.

[22] Wigner, "Unreasonable Effectiveness of Mathematics," p. 7.

[23] Analytic functions are those which are highly amenable to the calculus of Newton and Leibniz. They are smooth, meaning that their derivatives will yield meaningful results. This is vital in quantum mechanics because many of the observables, such as momentum, are indeed derivatives.

[24] Wigner, "Unreasonable Effectiveness of Mathematics," p. 7.

[25] Ibid., p. 8.

[26] Ibid., pp. 8-9.

[27] In this equation $G$ is the gravitational constant, which has a precise, contingent value, and $r$ is the distance between the objects whose masses are $m_1$ and $m_2$ respectively.

[28] The terms of this equation are as follows: $R_{\mu\nu}$ is the Ricci curvature tensor, $R$ is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, $\Lambda$ is the cosmological constant, $G$ is the gravitational constant from Newton's equation, $c$ is the velocity of light, and $T_{\mu\nu}$ is the stress-energy tensor. The left-hand side of the equation describes the curvature of spacetime and the right-hand side the mass-energy density.

[29] Viz., certain pairs of variables in Heisenberg's theory did not commute, that is to say, the calculations gave a different answer when done in reverse. It is this behavior which gave rise to his famous uncertainty principle because the first measurement "interferes" with the second. On the sole grounds of the fact that matrix manipulation is likewise non-commutative, Born and Heisenberg proposed to use matrices in Heisenberg's equations, with surprising success!

[30] Wigner, "Unreasonable Effectiveness of Mathematics," p. 9.

[31] Ibid., p. 10.

[32] Ibid.

[33] Ibid.

[34] Steiner, *Applicability of Mathematics*, p. 46.

[35] Mark Steiner, "Mathematics–Application and Applicability," in *Oxford Handbook of Philosophy of Mathematics and Logic*, ed. Stewart Shapiro, Oxford

Handbooks in Philosophy (Oxford: Oxford University Press, 2005), p. 631.

[36] Ibid.

[37] For example, Steiner thinks that the applicability of a mathematical concept can be explicable by a physical structure in the world. So, he writes, "To eliminate the mystery of a *particular* mathematical concept describing a *particular* phenomenon, we match the concept to a nonmathematical property, as before with linearity" (Steiner, *Applicability of Mathematics*, p. 44). Concerning the property of linearity, Steiner had written, "Whatever we are to say about this question, we can at least conclude this: there is no mystery concerning the applicability of linearity; the mathematical property of linearity can be reduced to physical properties which nature may either exhibit or not exhibit" (Ibid., p. 32). For Steiner. linearity, which applies whenever multiple solutions can combine by simple addition to provide further solutions, is simply a consequence of some natural behaviors' being the sum of many small parts. We shall take up below whether such isomorphism really resolves the mystery of applicability rather than merely shifts it; in any case Steiner provides abundant examples where such isomorphism does not hold.

[38] Steiner, *Applicability of Mathematics*, p. 45.

[39] Ibid., p. 73; cf. Steiner, "Mathematics–Application and Applicability," p. 631. So Steiner in fact takes Wigner' mystery to be a profound one. Unfortunately, Steiner imagines Wigner countering the above flaws by contending that his thesis applies to the set of mathematical concepts, not to the set of attempts to apply mathematical concepts, that is to say, it can be said of the mathematical concepts that a significant number of them prove significantly effective. This friendly advice strikes me as altogether wrong-headed. Wigner nowhere tries to assess what proportion of scientific attempts to apply mathematical concepts have proved successful (such a concern would be more relevant to Steiner's discoverability argument). And it would be inept to try to determine that a significant number of the untold infinity of infinities of mathematical concepts find application to the physical world. Rather Wigner may be understood, not to be focusing on isolated examples, but to be using a few examples merely to illustrate for his reader the general truth that mathematical concepts permeate physical science and have been singularly successful in describing the phenomena.

[40] As Peter Simons remarks, "The natural *Weltanschauung* for [the anthropocentric] view is theistic, with *Homo sapiens* as the chosen species" (Peter Simons, Critical notice of *The Applicability of Mathematics as a Philosophical Problem* by Mark Steiner, *British Journal for the Philosophy of Science* 52/1 [2001]: 182).

[41] *Pace* critics (e.g., Grattan-Guiness, "Solving Wigner's Mystery"), who interpret "unreasonable" to mean "irrational" and would refute Wigner by arguing that it is not irrational to think that mathematics should be effective in science. I think it is clear that

for Wigner belief in the effectiveness of mathematics in physics is not irrational, since he himself believed in it, but rather surprising or unexpected.

[42] Contrast Steiner's later rendition in Steiner, "Mathematics–Application and Applicability," p. 631. There he says more accurately that Wigner argues that (i) mathematical concepts are subject primarily to criteria internal to the mathematical community and (ii) in physics the reliance on mathematical concepts in formulating the laws of nature has led to laws of unbelievable accuracy.

[43] Even those who maintain that mathematics plays an explanatory role in science (e.g., Manfred R. Schroeder, "The Unreasonable Effectiveness of Number Theory in Physics, Communication and Music," *Proceedings of Symposia in Applied Mathematics* 46 [1992]: 1-19, http://dx.dol.org/10.1090/psapm/046/1 195839; Alan Baker, "Are there Genuine Mathematical Explanations of Physical Phenomena?" *Mind* 114/454 [2005]: 223-238; Marc Lange, "What Makes a Scientific Explanation Distinctively Mathematical?," *British Journal for the Philosophy of Science* 64/3 [2013]: 485-511) recognize that such is a case of acausal explanation. It amounts to nothing more than the broadly logical necessity of mathematical truths, e.g., Mother cannot divide 23 strawberries among her 3 children evenly because 23 is not divisible by 3 without remainder. Lange writes, "these explanations explain not by describing the world's causal structure, but roughly by revealing that the explanandum is more necessary than ordinary causal laws are. . . . These necessities are stronger than causal necessity, setting distinctively mathematical explanations apart from ordinary scientific explanations" (Lange, "What Makes a Scientific Explanation Distinctively Mathematical?," p. 491).

[44] Wigner, "Unreasonable Effectiveness of Mathematics," p. 14.

[45] Ibid., p. 2.

[46] Mary Leng, *Mathematics and Reality* (Oxford:  Oxford University Press, 2010), p. 239.

[47] Balaguer, *Platonism and Anti-Platonism in Mathematics*, p. 136.

[48] Wigner, "Unreasonable Effectiveness of Mathematics," p. 7.

[49] One must be wary in this connection of committing the fallacy of the Anthropic Principle, reasoning that the applicability of mathematics is the result of a self-selection effect, since we could not observe worlds that are not mathematically describable. As J. L. Mackie explains in another context, "This is not a good reply. There is only one actual universe, and it *is* therefore surprising that the elements of this unique set-up are just right for life when they might easily have been wrong. This is not made less surprising by the fact that it had not been so, no one would have been here to be surprised" (J. L. Mackie, *The Miracle of Theism* [Oxford: Oxford University Press, 1974], p. 141). The Anthropic

Principle requires for its legitimate employment the postulation of world ensemble, in this case featuring worlds varying in their mathematical describability, in which our universe appears.

[50] *Timaeus* 3-4.

[51] Peter van Inwagen, Richard Swinburne, and Keith Yandell come to mind. For a discussion of competing views on this issue see *Beyond the Control of God? Six Views on the Problem of God and Abstract Objects,* ed. Paul Gould, with articles, responses, and counter-responses by K. Yandell, S. Shalkowski, R. Davis, P. Gould, G. Oppy, and G. Welty (Bloomsbury: 2014).

[52] This is the option advocated by Alvin Plantinga and defended most extensively by Greg Welty. See Alvin Plantinga, "Theism and Mathematics," *Theology and Science* 9/1 (2011): 27-33; Alvin Plantinga, *Where the Conflict Really Lies: Science, Religion, and Naturalism* (Oxford: Oxford University Press, 2011), pp. 284-86; Greg Welty, "Theistic Conceptual Realism," in *Beyond the Control of God?: Six Views on the Problem of God and Abstract Objects*, edited by Paul M. Gould (London: Bloomsbury, 2014), pp. 81–96.

[53] Thomas V. Morris and Christopher Menzel, "Absolute Creation," *American Philosophical Quarterly* 23/4 (1986): 353–62.

[54] Wigner, "Unreasonable Effectiveness of Mathematics," p. 8.

[55] "On the Foundations of Physics," July 5, 2013, http://www.3ammagazine.com/3am/philosophy-of-physics/.

[56] Albert Einstein thought so: "One should expect a chaotic world which cannot be grasped by the mind in any way. One could (yes *one should*) expect the world to be subjected to law only to the extent that we order it through our intelligence. . . . By contrast, the order created by Newton's theory of gravitation, for instance, is wholly different. Even if the axioms of the theory are proposed by man, the success of such a project presupposes a high degree of ordering of the objective world, and this could not be expected *a priori*. That is the 'miracle' which is being constantly reinforced as our knowledge expands" (Albert Einstein, letter to Maurice Solovine, March 30, 1952, in Albert Einstein, *Letters to Solovine*, with an Introduction by Maurice Solovine, trans. Wade Baskin [New York: Philosophical Library, 1987]. pp. 132–133). I am indebted to Melissa Cain Travis for this reference.

[57] Steiner, *Applicability of Mathematics,* pp. 15, 27. See also Nancy Cartwright, *How the Laws of Physics Lie* (Oxford: Clarendon Press, 1983), p. 5, who argues that

mathematical physics cannot be regarded as directly describing the true structure of the physical world.

[58] The turning point from examples of descriptive applicability which Steiner deems not mysterious because they can be explained in terms of physical properties of nature to examples of descriptive applicability which do seem mysterious because they have no physical basis, occurs at pp. 35-36 of Steiner's book. For discussion of the following examples, see Steiner, *Applicability of Mathematics,* pp. 36-40, 95-97, 102.

[59] *Defending the Axioms: On the Philosophical Foundations of Set Theory* (Oxford: Oxford University Press, 2011), pp. 89-90.

[60] *On the Creation of the World* 16-20; 24.